401. Multiply out the top and split up the fraction.

402. Use the factor theorem, including a constant scale factor in the y direction.

403. A fraction is only zero if its numerator is zero. If the fraction is in its lowest terms, then its roots are exactly the roots of its numerator.

404. Consider $x = \pm\sqrt{3}$.

405. Rearranging to make p the subject is the same thing as solving for p. Use the quadratic formula for $\sqrt{p}$, then square both sides.

406. Replace x by $x - 4$.

407. After performing the calculation, trade factors of 10 between the two parts of the standard form.

408. Factorise the equations and consider the nature of the roots. A double root corresponds to a point of tangency.

409. The statement is phrased in terms of implication, so "and" is wrong.

410. Use *suvat*, taking g to be 10.

411. Find the surface area in terms of x, as three pairs of rectangular faces. Equate this to 88 and solve.

412. Consider the unit circle or a graph.

413. Enclose the triangle in a rectangular box, whose sides are parallel to the x and y axes. Subtract the areas of the triangles around the outside from the area of the rectangle.

414. Find $\mathbb{P}(X \cap Y \cap Z')$ and similar. You might find a Venn diagram useful.

415. To complete the square, take out a factor of a. You want an expression containing $(x + {}^{b}/_{2a})^2$.

416. Unfold the two faces to form a flat rectangle.

417. Remember that $(x - a) \equiv -(a - x)$.

418. Multiply out and compare coefficients.

419. (a) Compare $\dfrac{dy}{dx} = \lim_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$.
   (b) Consider the fact that the derivative of the function f is constant.
   (c) Simplify the top and cancel a factor of h.

420. One revolution is 360°, which is 2π radians.

421. Consider the input and the output transformations separately. For the input transformation, $(x - a)$ has been replaced by $(x - b)$; consider this as x being replaced by $(x - ...)$.

422. (a) You should get zero.
   (b) Use the factor theorem.

423. Consider the restriction of the possibility space by the given information.

424. (a) The domain is the set of inputs of a functional instruction. The same instruction can be given with many different domains, depending on the inputs under consideration.
   (b) The codomain is a coverall set whose purpose is to describe the type of output emerging from the function. It doesn't have to be precise.
   (c) The range is the specific set of outputs which are attainable with the chosen domain.

425. 0! is defined to be 1. Find the values of $\sin 0$ and $\cos 0$ either using the unit circle or graphs.

426. Solve the inequality $X^2 < X$ in the usual way, by solving the boundary equation and considering the behaviour of $X^2 - X$ either side of the roots you find. Then consider $[-2, 2]$, which has length 4, as the possibility space; find the size of the successful subset as a fraction of 4.

427. The region is a rectangle.

428. Write the sums out longhand.

429. Use the binomial expansion.

430. Turn the worded statements into algebra, and eliminate t.

431. Factorise as $(8 \cdot 2^x - ...)(2^x - ...) = 0$.

432. (a) The assumptions relate to
      i. the string and the pulley
      ii. just the string.
   (b) Each mass should have two forces acting on it.
   (c) Set up NII for each diagram, with resultant force taken in the direction of the acceleration. Then add the two equations to eliminate T.
   (d) Such problems are easiest to visualise if you extremify the situation: make the pulley very rough indeed, such that it barely moves.

433. Use an index law.

434. Assume, for a contradiction, that quadrilateral Q has four acute interior angles.

435. These are both straight lines through $(3, 2)$.

436. "Annular" means shaped like a ring.

437. In geometry, "normal" means "perpendicular to".

438. (a) $\Delta = 8^2 - 4 \cdot 2p \cdot (2p + 15)$. Simplify this.
   (b) You are told that the original quadratic has exactly one root, so set $\Delta = 0$. Solve this by factorising.

439. Rearrange to make y the subject. Compare to $xy = 1$, particularly for negative x.

440. Square brackets are inclusive [they include the "corners"], while round brackets are not inclusive (they don't include the "corners").

441. Sketch the curve.

442. (a) Add the forces.
   (b) Use Pythagoras.
   (c) Subtract the forces.

443. Such an equilateral triangular prism has three square faces and two equilateral faces.

444. Solve simultaneously by eliminating $(y + 2)^2$.

445. Consider the quadrilateral $XAYC$.

446. Use the factor theorem.

447. Without loss of generality, say that the first roll produced a six. Consider the second roll.

448. (a) Displacement is the integral of velocity.
   (b) Carry out the integral, or use *suvat*.

449. This is an input (x) transformation. Consider the effect on $y = \mathrm{f}(x)$ of replacing x by kx. There is no scaling in the y direction.

450. Use the factor theorem.

451. The vectors $\mathbf{i}$ and $\mathbf{k}$ are unit vectors in the x and z directions, and are therefore perpendicular. So, you can use Pythagoras to find the magnitude.

452. It makes no difference what the value of k is. So, sketch the problem for a particular value of k.

453. Add all three equations together.

454. Substitute $\mathrm{g}(x) = ax + 2$, then solve for a.

455. (a) The periods of sin and tan are $360°$ and $180°$. These are in x. Transform them according to the relevant input transformations.
   (b) Consider the lowest common multiple of the individual periods.

456. For a polynomial, a minimum must be a stationary points, i.e. the gradient must be zero.

457. Differentiate $y = x^3$. Substitute $x = 6/5$ to find the gradient of the tangent. The negative reciprocal is the gradient of the normal. Use $y - y_1 = m(x - x_1)$ for the normal, and sub $x = 0$.

458. Use 3D Pythagoras.

459. Integrate $\mathrm{f}'(x) = m$.

460. Use the factor theorem.

461. (a) There are six outcomes, so six points at which the branches of the tree end.
   (b) The successful outcomes are HH and THH.

462. Consider the effect on the point (p, q) of reflecting in $y = x$.

463. Set up and solve two simultaneous equations, NII horizontally and NII vertically.

464. A regular n-gon can be split up into $n - 2$ triangles.

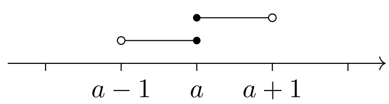
465. Treat the board as the possibility space.

466. Calculate $\frac{\Delta y}{\Delta x}$ and simplify.

467. A counterexample here is any two functions f and g which are not identical, but do have identical derivatives.

468. (a) Use $\frac{1}{2}ab \sin C$.
   (b) Solve for two possible angles, one of which is $\theta = \arcsin \frac{120}{169}$. Plug these into the cosine rule.

469. On a number line, the intervals are



470. (a) Speed u at θ above the horizontal resolves into components $u \cos\theta$ and $u \sin\theta$.
   (b) Set up a vertical *suvat*. Use the value from part (a), free-fall acceleration $a = g$, and final velocity $v = 0$.

471. Assume, for a contradiction, that f is quadratic, so $f(x) = ax^2 + bx + c$. Differentiating twice gives $f''(x) = 2a$, which is constant. Show that $a = 0$ and find a contradiction from there.

472. Even though the cards are picked out together, you can still consider them one by one.

473. Consider the boundary values $x = a$ and $x = b$ as possible counterexamples to the implications.

474. Substitute $x = -2/3$ and solve for p. Then solve for q.

475. Constant factors can be taken out of integrals, and sums can be split up.

476. This is a quadratic in x^2.

477. (a) Add the first five integers.
   (b) Adding the sums gives n copies of the same quantity.

478. (a) Adjacent circles intersect each other at right angles. Find the distance between the centres of two adjacent circles. Equate this distance to two radii minus the overlap, and rearrange.
   (b) The smallest square is arranged obliquely. So, calculate the width of the group of circles, in a NW-SE direction.

479. Set angle $C = 90°$ in the cosine rule.

480. Switch x and y, then add 5 to y.

481. Multiply out and take out a common factor of two. Be careful to note why the other factor must be an integer.

482. Take out a common factor of x first, leaving a quadratic in x^2.

483. Multiply out the brackets before integrating, and remember the constant of integration.

484. Divide top and bottom by x. Then consider the limit of the small fractions $1/x$ and $-11/x$.

485. Find the equation of the hypotenuse, and therefore the equation of OP. Solve simultaneously.

   Alternatively, use similar triangles.

486. Test $x = \pm a$. Use the factor theorem to take out the relevant factors. Factorise fully and identify the squared factor, and thus the double root.

487. Write both quantities in terms of edge length l, then eliminate l.

488. Negative numbers have real cube roots, but do not have real square or fourth roots.

489. Multiply the identity by $(x-4)$, then write the LHS in expanded polynomial form. Equate coefficients.

490. In each case, test $x = \pm 1$.

491. The distance of a point from a line is defined as the shortest distance from the point to a line. This is the distance along a perpendicular dropped to the line.

492. On the graph of a normal distribution, as with any continuous distribution, area is probability. So, in each case, identify the relevant area and write down the probability. Note that one of the answers is zero.

493. Carry out the definite integration. Find a root using a polynomial solver or a numerical method. Take the relevant factor out and then consider Δ in the remaining quadratic.

494. Work out the distances each ship has travelled. Draw a clear diagram of the positions. Use the cosine rule to find the distance $|AB|$, then use the sine rule to find $\angle PBA$. Note that bearings are always given clockwise from north.

495. Draw a Venn diagram.

496. Write x in terms of w, and substitute.

497. Find the magnitude of the acceleration as the size of the hypotenuse of a right-angled acceleration triangle. You have one of the other sides.

498. The first graph is a standard exponential curve. The second graph is a reflection of the first.

499. One answer is yes, two are no. In the case of yes, the reason is that the new LHS can be thought of as a translation of the old LHS.

500. Substitute in the values $t = 0$ and $t = \sqrt{2}$ to find the coordinates of the endpoints of the interval. Then use Pythagoras to find the distance between the endpoints.

——— End of 5th Hundred ———